Algebra II — Dr. Paul L. Bailey

Homework 1116 — Tuesday, November 16, 2021

Name:

Due Tuesday, November 16, 2021. Write all complex numbers and polynomials in standard form. Do not copy. Do not write anything you do not understand.

Definition 1. The *end behavior* of a polynomial f is one of the following:

- $\nwarrow\nearrow$ if f is positive on the far left and positive on the far right (or +|+)
- $\swarrow\nearrow$ if f is negative on the far left and positive on the far right (or -|+)
- $\nwarrow\searrow$ if f is positive on the far left and negative on the far right (or +|-)
- $\swarrow\searrow$ if f is negative on the far left and negative on the far right (or -|-)

Proposition 1. *Let $f(x) = a_n x^n + \cdots + a_1 x + a_0$ be a general polynomial of degree n (in standard form). The end behavior of f can be determined by the degree n and the leading coefficient a_n, using this chart.*

	n even	*n odd*
$a_n > 0$	$\nwarrow\nearrow$	$\swarrow\nearrow$
$a_n < 0$	$\swarrow\searrow$	$\nwarrow\searrow$

Problem 1. Determine the end behavior of each of the following polynomials.

(a) $x^2 - 25$

(b) $8x - 3x^2$

(c) $x^3 - 7x + 5$

(d) $(x-1)(x-3)^2(x-5)^3$

(e) $x^5 - x^6$

(f) $2x$

Problem 2. Solve the equation $6x^2 + 13x - 5 = 0$. Simplify the solutions. Write the solution set.

Problem 3. Consider the polynomial $f(x) = x^3 - 8x^2 + 21x - 18$.
Find the multiplicity the following numbers as zeros of f.

Number	1	2	3	4	5	6
Multiplicity						

Problem 4. Let f be the unique monic polynomial with real coefficient such that $f(3+5i) = 0$. Write $f(x)$ in standard form.

Problem 5. Let $f(x)$ be the unique monic polynomial of minimal degree with zeros -3, 5, and 7. Write $f(x)$ in standard form.

Remark 1. Recall that $(x - r_1)(x - r_2) = x^2 - (r_1 + r_2)x + r_1 r_2$.

Problem 6. Let $f(x) = (x - r_1)(x - r_2)(x - r_3)$. Multiply out this polynomial to find its coefficients.
That is: suppose we write the same f as $f(x) = ax^3 + bx^2 + cx + d$.
Find a, b, c, and d in terms of r_1, r_2, and r_3.
Do not skip this problem because it is "too hard". It is not "too" hard.
Just do it; you will learn something.